TRANSIENT THERMAL STRESS ANALYSIS OF A PIPE JUNCTION

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THESIS

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ABSTRACT

A method is devised for the transient-state thermal stress analysis of two pipes, joined butt to butt and subjected to rapid or sudden change of internal fluid temperature. Although it is assumed that there is symmetry about the common pipe axis, the properties of the materials as well as the thickness of each pipe may be different. Then, given a specified time-temperature relationship for the internal fluid, over a specified problem time, the maximum stress encountered may be obtained. A digital computer program is appended for the solution of such problems.

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TABLE OF SYMBOLS

English Letters

Symbol	
a	inner radius of pipe
b	outer radius of pipe
c	specific heat
D	plate flexural rigidity = $Eh^3/12(1-\mu^2)$
E	Young's modulus of elasticity
G	sum of errors for least squares analysis
h	pipe thickness
h_1	coefficient of surface heat transfer
K	thermal conductivity
M	bending moment (per unit circumference)
M_o'	dislocation bending moment (per unit circumference) at pipe junction
M_o	total bending moment (per unit circumference) at pipe junction = $M_o' - M_\infty$
M_∞	bending moment (per unit circumference) remote from pipe junction
N	number of data points in thermal transient analysis
r	radial coordinate with origin at center of pipe
r_m	$\frac{a+b}{2}$
t	time
T	temperature
u	radial displacement, remote from junction, positive outward

V	volume, also radial force (per unit circumference)
V_o	radial force (per unit circumference) at pipe junction
w	axial displacement, measured positive away from pipe junction
y	radial distance, measured positive outward from mid-surface of pipe
z	axial coordinate with origin at pipe junction

Greek Letters

α	coefficient of linear thermal expansion
β	$\sqrt[4]{\frac{3(1-\mu^2)}{r^2h^2}}$
γ	Fourier modulus = $\frac{k\,\Delta t}{c\,\rho\,V}$
ε_r	unit elongation in radial direction
ε_t	unit elongation in circumferential direction
ε_z	unit elongation in axial direction
η	radially outward deflection due to dislocation stress only
θ	tip slope = $d\eta/dz$
μ	Poisson's ratio
ρ	weight density of pipe material
$\sigma_r, \sigma_t, \sigma_z$	thermal (radial, circumferential, axial) stress remote form pipe junction
σ_{tz}, σ_{zz}	dislocation (circumferential, axial) stress
$\sigma_{T_r}, \sigma_{T_t}, \sigma_{T_z}$	total (radial, circumferential, axial) stress
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
σ_{sig}	maximum significant stress
τ_{Trz}	shearing stress

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I. INTRODUCTION

In the design and analysis of certain piping of critical importance, such as nuclear power piping, approved methods, such as those in the USAS B31.7 Code, call for analysis of transient thermal stresses. The most important cause of such transient thermal stresses is variation of the temperature of the fluid flowing in the piping. It is conventional to regard the fluid as having infinite heat capacity and to presume that the fluid flows through the piping with sufficient rapidity that all parts of the piping component or assembly in question are simultaneously exposed to the same interior ambient temperature. At any rate, an analysis based upon such simplifying assumptions may provide the engineering answer that is required or may form the basis for a more precise evaluation.

The geometry of piping components and assemblies is generally rather complicated and one method of analysis is to study a greatly simplified geometry and then to apply "stress indices" which are tabulated constants intended to reflect the greater severity of the thermal stresses in the actual object as compared to that having the idealized geometry. One such simplified geometry of importance is that of two semi-infinite pipes, having a common axis, and joined together by welding or brazing at the origin of an axial coordinate system. If the two pipes have identical dimensions and properties, determination of the transient thermal stresses is a rather simple problem: there is no variation along the axial length. However, if the two pipes do not match in thickness or in material properties, so-called dislocation stresses will be set up near the junction and the task of determining the significant transient thermal stresses becomes considerably more difficult. It may be noted that the analysis to be given herein also contemplates a mismatch in pipe diameter, but this is not important since the theory employed will be limited to the case of thin-wall pipes.

The problems of interest arise upon considering the ambient fluid temperature, to which the interior of the piping is convectively exposed, as varying with respect to time in a specified fashion. In the development which follows, the general approach is capable of dealing with any temperature-time relation. The implementation presented is in the form of an immediately useful digital computer program for the analysis of these problems. In addition, there is specialization to temperature-time relations which postulate a long period at constant temperature followed by a sudden change in temperature or by a rapid linear variation in temperature which is in turn followed by maintenance of a new steady temperature.

The analysis of the thermal stresses in any case consists of the following steps:

(1) determination of the temperature distribution in the piping

(2) determination of the thermal stresses in the piping at a considerable distance from the junction

(3) determination of the dislocation reactions, i.e., the radial force and bending moment (per unit circumference) exerted by one of the semi-infinite pipes upon the other, and the calculation of stresses resulting therefrom

(4) combining these stresses in an appropriate manner to find what could be called the "significant stress" and locating the point in the piping and the point in time for which this significant stress is greatest

The task is greatly simplified because the problem at hand is one possessing symmetry about the common pipe axis. It is assumed that the fluid temperature is the same function of time at all points adjacent to the inner wall of the pipe and that the coefficient of convective heat transfer is likewise the same at all such points. It is also assumed that the exterior surfaces of both pipes are perfectly insulated and that the following properties of both pipes are given and classified into two categories: first, thermal conductivity, specific heat, and density which may vary with temperature; and second, Young's modulus of elasticity, Poisson's ratio, and the coefficient of linear thermal expansion which must remain constant. Under certain conditons the properties in the second category might be permitted to vary, but this has not been investigated in this thesis. It is also necessary to know pipe wall thickness and outside diameter.

The task of determining the transient temperature distribution in each of the two pipes is simplified by the assumption that there is no axial flow of heat even though adjacent points in the two pipes may not be at the same temperature. In other words, an error is introduced by neglecting axial flow of heat near the junction but the problem is too difficult to handle without this assumption. There is no evident way to estimate the effect of the assumption but the effect should be small for thin wall pipes to which this entire development is aimed.

With this assumption, the determination is relatively straightforward. If the properties of the materials were not themselves dependent upon temperature, there might be some incentive to seek analytic solutions. However, for the sake of generality, a numerical method has been used. This consists of slight modifications to a digital computer program, BETTY1, developed by John E. Brock, Professor of Mechanical Engineering, Naval Postgraduate School, Monterey, California. This program is based upon the general algorithm described in Reference 5, but including several auxiliary features; see Appendix B.

The output of BETTY1 is a tabulation, at each instant in time, of the temperature at equally spaced radii through the pipe wall. In the present application this provides too much data to handle so that the next step is to pass a polynomial approximation of chosen degree (usually of third or fourth degree) through the data generated by BETTY1, and retain the coefficients of this polynomial for each instant of time. This also provides for more convenient processing at later stages of the analysis.

Next, using this information, the stress distribution and radial displacement is calculated for each pipe at a position remote from the junction. This is a simple and standard sort of calculation which is simplified by virtue of the temperature data being given in polynomial form. Here, and elsewhere in what follows such calculations must be made for each pipe at each instant in time.

The next step is the determination of the dislocation reactions, i.e., the radial force and bending moment per unit circumference at the pipe junction. This calculation is based upon the fact that continuity of radial deformation and its first derivative with respect to the axial coordinate must be maintained at the junction. It leads to a pair of simultaneous equations in which the results of the calculation described in the preceding paragraph enter. Having the dislocation reactions, the corresponding dislocation stresses may be determined. These all contain a negative exponential term and die out away from the junction. However, they are not necessarily maximum right at the junction, so that some care is required later to be sure that the worst condition is not overlooked. The actual stress components at any point are obtained by superposing the thermal stresses remote from the junction and the dislocation stresses near the junction.

Finally it is necessary to determine the "significant stress" or measure of stress severity. In conformity with the stress criteria used by the most applicable codes, the maximum shearing stress criterion has been employed here. At any cross section, at any radius, and at any time, this maximum shearing stress[1] may be calculated without great difficulty. Because the variation with respect to radius occurs only in the non-dislocation stresses (except for a linear variation) and because the variation with respect to axial position occurs only in the dislocation stresses, the problem of determining a maximum shearing stress is simplified.

In brief, the analysis and its implementation in the form of a digital computer program called PAM1 perform all the above steps and emerge with a statement of the magnitude of the "most significant stress," its location, and the time at which it occurs.

The effort in developing this thesis has been in understanding and relating the several steps as described above, in writing sub-programs to perform the operations in each of the steps as outlined above, making use of programs (such as BETTY1 and a least squares curve fitting routine) that appeared to be of immediate use, debugging and proving each portion, knitting the separate parts together into a

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[1] It is conventional to calculate and employ a quantity known as the "significant stress" or the "stress intensity" which is equal to twice the maximum shearing stress. This practice is adhered to in what follows herein.

complete whole, and finally debugging and proving the final program.

The steps of the theoretical analysis and the methods devised to employ them are described in Sections II and III hereof. The digital computer program is described, in general terms, in Section IV. Instructions for the use of the program are given in Appendix A. The program listing is given in Appendix B.

There was not sufficient time after developing the program to make a systematic use of it to investigate the effect of certain material or geometrical parameters upon the value of the significant stress. It is recommended that this become the point of departure for a future thesis.

II. THEORY

When the wall of a pipe is nonuniformly heated as for instance when it is subjected to a rapid or sudden change of internal fluid temperature, its elements do not expand uniformly and mutual interference sets up thermal stresses. It is assumed that the distribution of the temperature does not vary along the axis of the pipe and that there is complete symmetry about the axis. A ring is cut from the pipe by two plane cross sections perpendicular to the axis and a unit distance apart. During thermal deformation such cross sections can be assumed to remain plane if taken sufficiently distant from the ends of the pipe. Therefore, the unit elongation in the direction of the axis is constant [7]. Then, the unit elongations in the perpendicular directions are

$$\varepsilon_z = \frac{dw}{dz} = \text{constant}$$

$$\varepsilon_r = \frac{du}{dr} \qquad (1)$$

$$\varepsilon_t = \frac{u}{r}$$

These elongations can be expressed as functions of the axial, radial, and circumferential stresses remote from the end of the pipe (σ_z, σ_r, and σ_t respectively), the thermal expansion, α, and the temperature, T, where T varies with the radial distance, r, only.

From Reference 6, p. 66, the elongations are

$$\varepsilon_z = \frac{\sigma_z}{F} - \frac{\mu}{F}(\sigma_r + \sigma_t) + \alpha T$$

$$\varepsilon_r = \frac{\sigma_r}{F} - \frac{\mu}{E}(\sigma_z + \sigma_t) + \alpha T \qquad (2)$$

$$\varepsilon_t = \frac{\sigma_t}{F} - \frac{\mu}{E}(\sigma_z + \sigma_r) + \alpha T$$

Let Δ be the increase in unit volume. From this

$$\Delta = \varepsilon_z + \varepsilon_r + \varepsilon_t = \frac{1 - 2\mu}{E}(\sigma_z + \sigma_r + \sigma_t) + 3\,\alpha T \qquad (3)$$

Substituting equations (2) into equation (3) it can be shown that

$$\sigma_z = \frac{E}{1+\mu}\left(\varepsilon_z + \frac{\mu}{1 - 2\mu}\Delta\right) - \frac{\alpha TE}{1 - 2\mu}$$

$$\sigma_r = \frac{F}{1+\mu}\left(\varepsilon_r + \frac{\mu}{1 - 2\mu}\Delta\right) - \frac{\alpha TE}{1 - 2\mu} \qquad (4)$$

$$\sigma_t = \frac{E}{1+\mu}\left(\varepsilon_t + \frac{\mu}{1 - 2\mu}\Delta\right) - \frac{\alpha TE}{1 - 2\mu}$$

From Peference 7, p. 206, the equation for radial equilibrium of an element is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0 \qquad (5)$$

Substituting equations (4) and (1) into equation (5)

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{1 + \mu}{1 - \mu}\,\alpha\,\frac{dT}{dr} \qquad (6)$$

which may be written in the form

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru)\right] = \frac{1+\mu}{1-\mu}\,\alpha\,\frac{dT}{dr} \tag{7}$$

Integration with respect to r gives

$$\frac{d}{dr}(ru) = \frac{1+\mu}{1-\mu}\,\alpha Tr + 2C_1 r \tag{8}$$

A second integration gives

$$u = \frac{1}{r}\,\frac{1+\mu}{1-\mu}\int_a^r \alpha Tr\,dr + C_1 r + C_2\,\frac{1}{r} \tag{9}$$

where C_1 and C_2 are constants of integration. C_1 and C_2 are found from the boundary conditions at the inner and outer surfaces of the pipe, namely

$$(\sigma_r)_{r=a} = 0 \; ; \quad (\sigma_r)_{r=b} = 0 \tag{10}$$

From equations (10) the constants of integration are evaluated as

$$C_2 = \frac{1+\mu}{1-\mu}\,\frac{a^2}{b^2-a^2}\int_a^b \alpha Tr\,dr$$

$$C_1 = \frac{(1+\mu)(1-2\mu)}{(1-\mu)}\,\frac{1}{b^2-a^2}\int_a^b \alpha Tr\,dr - \mu\varepsilon_z \tag{11}$$

An expression for σ_r is obtained by substituting $\varepsilon_r = du/dr$ and $\varepsilon_t = u/r$ into the second equations (4) and then taking u from equation (9) and the values of the constants from equations (11). This gives

$$\sigma_r = \frac{E}{1-\mu}\left[-\frac{1}{r^2}\int_a^r \alpha Tr\,dr + \frac{r^2-a^2}{r^2(b^2-a^2)}\int_a^b \alpha Tr\,dr\right] \quad (12)$$

An expression for σ_t is obtained from the equation of equilibrium (5), thus

$$\sigma_t = \sigma_r + r\frac{d}{dr}\sigma_r$$

From this,

$$\sigma_t = \frac{E}{1-\mu}\left[\frac{1}{r^2}\int_a^r \alpha Tr\,dr + \frac{r^2+a^2}{r^2(b^2-a^2)}\int_a^b \alpha Tr\,dr - \alpha T\right] \quad (13)$$

An expression for σ_z is obtained by substituting $\varepsilon_r = du/dr$ and $\varepsilon_t = u/r$ into the first of equations (4) and then taking u from equation (9) and the values of the constants from equation (11). This expression will contain the constant elongation, ε_z, in the direction of the axis of the pipe. If it is assumed that the pipe can expand freely in the axial direction, the magnitude of ε_z can be calculated from the condition that the sum of the axial forces over any normal cross section is equal to zero. From this

$$\varepsilon_z = \frac{2}{b^2-a^2}\int_a^b \alpha Tr\,dr \quad (14)$$

and

$$\sigma_z = \frac{2E}{(1-\mu)(b^2-a^2)}\int_a^b \alpha Tr\,dr - \frac{E\alpha T}{1-\mu} \quad (15)$$

Substituting the values of the constants from equation (11), the radial displacement remote from the pipe junction is

$$u = -\mu r \varepsilon_z + \left(\frac{1+\mu}{1-\mu}\right)\left[\frac{1}{r}\int_a^r \alpha T r dr + \frac{(1-2\mu)r + \frac{a^2}{r}}{b^2 - a^2}\int_a^b \alpha T r dr\right] \quad (16)$$

Thus the thermal stresses, σ_r, σ_t, and σ_z, and radial displacement, u, at a distance from the pipe junction may be determined by use of equations (12), (13), (15) and (16). It should be noted here that at a distance remote from the pipe junction there is no variation of the stresses or radial displacement with respect to z, the axial coordinate of the pipe. There exists here a bending moment per unit circumference, denoted as M_∞, where

$$M_\infty = \int_a^b \sigma_z \left(r - \frac{a+b}{2}\right) dr \quad (17)$$

See Figure 1, case A

On the other hand, if the end of the pipe is supported or clamped, as for example when it is joined to another pipe, free expansion of the pipe is prevented and a bending moment per unit circumference, M_o' and a radial force per unit circumference, V_o, are set up at the pipe junction (Figure 1, case B). Because of the continuity with the joining pipe, the moment, stresses, and radial deflection at the pipe junction will be different from those at a distance from the end of the pipe. In other words, the adjoining pipe exerts a bending moment and radial force.

It should be noted here that the total bending moment at the pipe junction is $M_o' - M_\infty$, which will be represented here as M_o. Thus, in solving for the dislocation reactions, although M_o is obtained directly, it is worth noting that that bending moment depends on both the moment at the pipe junction as well as that remote from the junction. The total radial force is just that at the pipe junction, V_o, since there is no force remote from the junction (Figure 1).

This bending moment and radial force cause a deflection and tip slope at the end of the pipe (z = 0), which are denoted by η and θ, respectively. The same is true, however for the other. Therefore, let η_1, and θ_1, denote the deflection and tip slope, respectively, at the end of the adjoining pipe. Now if these pipes are to join up continuously, the total deflection for each pipe at the pipe junction must be the same. The total deflection consists of the deflection due to M_o' and V_o as well as the radial displacement, u, remote from the junction. Equating these total deflections, thus

$$\eta + u = \eta_1 + u_1 \tag{18}$$

Also, if these pipes are to join continuously, the tip slopes at the ends of each pipe must be negative of each other. That is,

$$\theta = - \theta_1 \tag{18a}$$

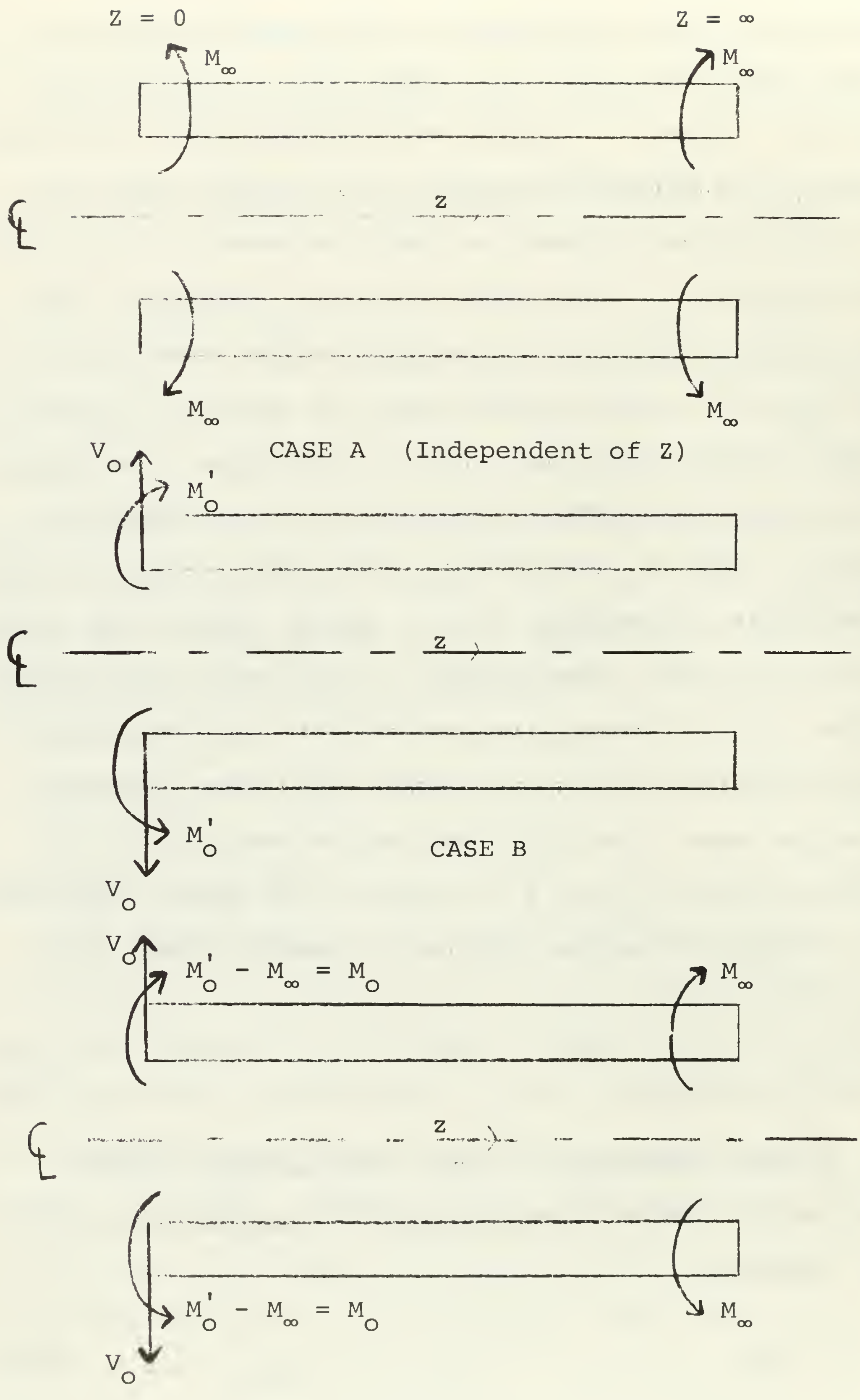

Figure 1. Combination of Cases A and B

From Reference 7, it is seen that

$$\eta = \frac{e^{-\beta z}}{2\beta^3 D} [\beta M_o' (-\sin \beta z + \cos \beta z) + V_o \cos \beta z] \qquad (19)$$

where

$$D = \frac{Eh^3}{12(1 - \mu^2)}$$

and

$$\beta^4 = \frac{3(1 - \mu^2)}{(\frac{a+b}{2})^2 h^2}$$

and where the algebraic signs have been changed to conform to the convention herein that η is positive outward. If the pipe junction is taken as the origin of the axial coordinate (i.e., z = 0), then the deflection at the junction is

$$\eta = \frac{1}{2\beta^3 D} [\beta M_o' + V_o] \qquad (20)$$

Also from Reference 7, the tip slope is the first derivative of the deflection with respect to z, or

$$\theta = \frac{d\eta}{dz} = -\frac{e^{-\beta z}}{2\beta^2 D} [2\beta M_o' \cos \beta z + V_o (\cos \beta z + \sin \beta z)] \qquad (21)$$

Therefore the tip slope at the pipe junction (z = 0) is

$$\theta = -\frac{1}{2\beta^2 D} [2\beta M_o' + V_o] \qquad (22)$$

It should be noted that the radial displacement, u, given in equation (16) is a function of the radius, r. The value of u used in equation (18) was the average value of the radial displacement found at the inner and outer radii of the pipe. This seems to be the most reasonable and convenient of several different choices that could be made.

Noting that β and D are functions of the material and the geometry of each pipe, their values may be obtained and substituted into equations (20) and (22). Also the values of u and u_1, may be obtained from equation (16). For mechanical equilibrium, we must have $M_o = M_o' - M_\infty = M_{o1} = M_{o1}' - M_{\infty 1}$ and $V_o = -V_{o1}$. These relations, together with equations (18) and (18a) permit solving for the dislocation reactions M_o', M_{o1}', and V_o. Hence, the dislocation stresses near the pipe junction can be obtained.

From Reference 7, the axial stress due to the bending moment is

$$\sigma_{zz} = \frac{-12yM(z)}{h^3} \tag{23}$$

where y is the radial distance measured from midsurface, so that $-\frac{h}{2} \leqq y \leqq \frac{h}{2}$. Also from Reference 7

$$M(z) = \frac{V_o}{\beta}\,[e^{-\beta z}\sin \beta z] + M_o'\,[e^{-\beta z}(\cos \beta z + \sin \beta z)] \tag{24}$$

The maximum axial stress is obtained at the axial position where M(z) is a maximum. Since M(z) contains a negative exponential term, it will eventually decrease to zero as z increases. But this does not necessarily mean that M(z) will be maximum at the origin. The method of finding the point of maximum bending moment is shown in detail in the next section.

From Reference 7 the circumferential stress due to the dislocation reactions is

$$\sigma_{tz} = \frac{E\eta(z)}{r_m} + \mu\sigma_{zz} \tag{25}$$

Where $\eta(z)$ comes from equation (19) and $r_m = \frac{a+b}{2}$. It should be noted here that there is no radial stress due to the dislocation reactions because of the assumption that the pipe wall is thin.

Summarizing now, there is a radial, circumferential, and axial stress remote from the junction as well as additional circumferential and axial stresses near the junction. Each of these is some function of the radius, so that for a given radius and given axial location these stresses may be superposed on each other and the total stresses found. Using equations (12), (13), (15), (23), and (25) these total stresses are defined as

$$\begin{aligned} \sigma_{T_r} &= \sigma_r \\ \sigma_{T_t} &= \sigma_t + \sigma_{tz} \qquad (26) \\ \sigma_{T_z} &= \sigma_z + \sigma_{zz} \end{aligned}$$

The leading subscript T indicates "total."

The radial force V, also gives rise to a shearing stress, τ_{Trz}. Presuming that this shearing stress is distributed parabolically, it can be represented as

$$\tau_{Trz} = \frac{3V}{2h} [1 - (\frac{2y}{h})^2] \tag{27}$$

where, from Reference 7

$$V = e^{-\beta z} [V_o (\cos \beta z - \sin \beta z) - 2\beta M_o' \sin \beta z] \tag{28}$$

Thus a general element is loaded as shown in Figure 2. (It should be noted that this element is also subjected to the circumferential stress, σ_{Tt}).

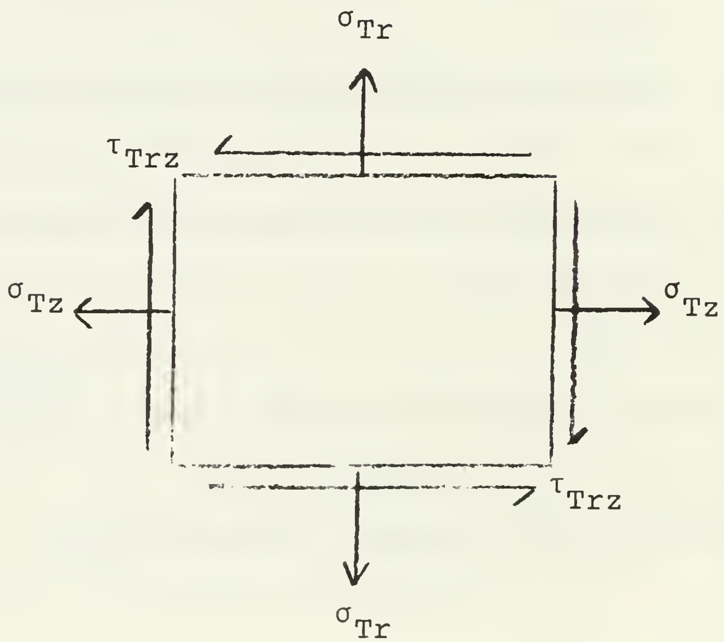

Figure 2

The three principal stresses are then

$$\sigma_1 = \sigma_{Tt}$$

$$\sigma_2 = \frac{\sigma_{Tz} + \sigma_{Tr}}{2} + \sqrt{\left(\frac{\sigma_{Tz} - \sigma_{Tr}}{2}\right)^2 + \tau_{Trz}^{\ 2}} \qquad (29)$$

$$\sigma_3 = \frac{\sigma_{Tz} + \sigma_{Tr}}{2} - \sqrt{\left(\frac{\sigma_{Tz} - \sigma_{Tr}}{2}\right)^2 + \tau_{Trz}^{\ 2}}$$

The maximum significant stress at a point is defined as

$$\sigma_{sig} = \text{Max}\ \{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\}. \qquad (29a)$$

It is necessary to find the point in space (i.e., values of r and z) and the point in time for which σ_{sig} achieves its maximum value, and to determine this value. This must be done for each of the two pipes individually since allowable values of σ_{sig} may be different for the two pipes. How this is done is considered in detail in Section III.

III. METHOD

As indicated in Section I, the program BETTY1 has been incorporated into the program PAM1 to obtain the transient temperature distribution. This is a program for dealing with the one dimensional thermal transient analysis of homogeneous slabs or tubes. Its use herein is for the thermal transient analysis of a homogeneous pipe in which the outer surface is perfectly insulated and the inner surface is convectively exposed to a fluid having a specified time-temperature relationship. Physical properties of the pipe material (i.e. thermal conductivity, weight density, and specific heat) may be temperature dependent; the program is set up to consider property-temperature relationships approximated by a third degree polynomial. A similar variation is permitted in the surface heat transfer coefficient.

The following is a brief summary of the general algorithm of the program [5]. As shown in Figure 3 a unit

Figure 3

length of pipe, contained in a small dihedral angle, is considered to be divided into subvolumes, where the inner and outer subvolume have half the thickness of the others. The thermal properties of each subvolume are considered to be concentrated at its central nodal point, and heat is imagined to be conducted between nodal points through a network of fictitious heat-conducting rods of appropriate thermal conductance. In the transient state, however, heat is not only conducted to and from each nodal point in the lumped network, but additionally each nodal point experiences a change in internal energy in an amount which depends upon its temperature change during a given time increment, its specific heat, the total volume of material which it represents, and the density of the material.

Assuming that the specific heat, c, and the thermal conductivity, K, are uniform, the heat conducted by each rod is $-K\Delta T$, where ΔT is the temperature difference between adjacent nodal points. The total heat conducted in the finite time increment Δt is $-K\Delta T\Delta t$. Now the change in internal energy of a given nodal point is $c\rho V\Delta T'$, where ρ is the material density, V is the subregion volume, and T' is the temperature of the nodal point at the end of the finite time interval Δt. The heat balance can be written as

$$-\Sigma K\Delta T\Delta t = c\rho V\Delta T' \qquad (30)$$

Using an example of three nodal points, as shown here

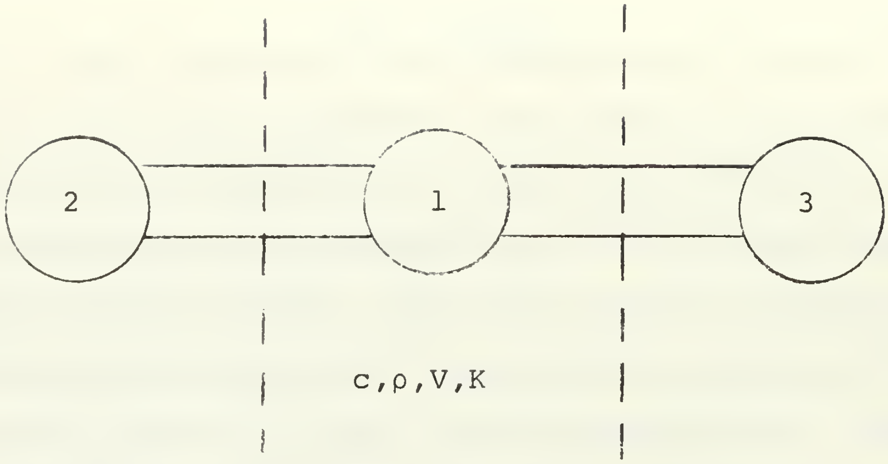

Figure 4

a specific example of equation (30) is

$$K(T_2 - T_1) + K(T_3 - T_1) = \frac{c\rho V}{\Delta t}(T_1' - T_1).$$

Solving for T_1', the future temperature of the nodal point,

$$T_1' = \gamma[T_2 + T_3 + (\frac{1}{\gamma} - 2)\ T_1] \quad (31)$$

where

$$\gamma = \frac{K\Delta t}{c\rho V}$$

and is known as the Fourier modulus. From this it can be seen that the future temperature of 1 depends on its present temperature and the present temperature of its adjacent nodal points. Equation (31) is the basis of the transient-state numerical method. It should be noted that the primary purpose of BETTY1 was to find the quantities delta-tee-one and delta-tee-two, defined and required by the Nuclear Power Piping code, but in the present application only the transient-state temperature distribution is of interest.

The immediately applicable accessible output from BETTY1 is a set of nodal point temperatures at each epoch of time. For convenience in what follows, it was decided to represent this temperature distribution by a polynomial in r, using least squares procedures. This results in an economy of computer storage and convenience in performing the integrations required in equations (9) and (11) (et.seq.), with minor loss of accuracy. A brief discussion of the curve fitting procedure follows.

The deviation at a data point is defined as the difference between the tabulated T-value and the T-value computed for a functional relationship. The least-squares criterion asks for a minimum of

$$G = \sum_{i=1}^{N} [T'(r_i) - T_i]^2 \tag{32}$$

where N equals the number of data points, T' denotes a value computed at $r = r_i$ from a functional relationship, and T_i denotes the corresponding tabulated value. This can be differentiated to find its minimum, which leads to equations that in many cases of practical interest are linear and, in principle at least, easy to solve.

The form of the relationship must be chosen in advance. Since it is important to find a relationship which will be easy to obtain and integrate, a polynomial of the form

$$T(r) = a_0 + a_1 r + a_2 r^2 + \ldots a_k r^k = \sum_{j=o}^{k} a_j r^j \tag{33}$$

was selected. Here k is the degree of the polynomial and the coefficients a_0, a_1, a_2 ... a_k must be determined to minimize G.

Substituting equation (33) into equation (32)

$$G = \sum_{i=1}^{N} \left(\sum_{j=0}^{k} a_j r_i^j - T_i \right)^2 \tag{34}$$

For a minimum, the partial derivative of G with respect to each coefficient, a_m, (m = 0, 1, 2 ..... k) must vanish. Thus

$$0 = \frac{\partial G}{\partial a_m} = \sum_{i=1}^{N} [2 \left(\sum_{j=0}^{k} a_j r_i^j - T_i \right) r_i^m]$$

$$= 2 [\sum_{j=0}^{k} a_j \left(\sum_{i=1}^{N} r_i^{j+m} \right) - \sum_{i=1}^{N} T_i r_i^m] \tag{35}$$

$$m = 0, 1, 2 \ldots k$$

This set of equations may be arranged as

$$\begin{bmatrix} N & \Sigma r_i & \Sigma r_i^2 \ldots\ldots \Sigma r_i^k \\ \Sigma r_i & \Sigma r_i^2 & \Sigma r_i^3 \ldots\ldots \Sigma r_i^{k+1} \\ & \vdots & \vdots \\ \Sigma r_i^k & \Sigma r_i^{k+1} & \Sigma r_i^{k+2} \ldots\ldots r_i^{2k} \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \Sigma T_i \\ \Sigma T_i r_i \\ \vdots \\ \Sigma T_i r_i^k \end{bmatrix} \tag{36}$$

where, in this last expression, Σ means $\sum_{i=1}^{N}$. It is straightforward to calculate the coefficients and to solve the system to obtain the desired coefficients a_m.

For the sudden or rapid temperature changes which appear to be of greatest interest, it seems likely that the degree of the polynomial need not exceed k = 4, although no verification of this conjecture has been undertaken. However, it should be noted here that even for some unusual time-temperature relationship such as a temperature which fluctuates with time, a polynomial of larger degree could still be used to represent this temperature distribution.

Now, for each pipe and for each time epoch, the coefficients of the polynomial representation of the temperature as a function of the radius are available. These polynomials can be integrated and the stresses remote from the pipe junction can be determined. Equation (12)

$$\sigma_r = \frac{E}{1-\mu}\left[-\frac{1}{r^2}\int_a^b \alpha T r dr + \frac{r^2 - a^2}{r^2(b^2 - a^2)}\int_a^b \alpha T r dr\right] \qquad (12)$$

will serve as an example here. Given values of E, μ, α, a, and b, and the particular radius, r, as well as the polynomial for the temperature distribution, the radial stress can be found. On a computer, the radius can be incremented from the inner radius, a, to the outer radius, b, thereby finding the radial stress at increments throughout the thickness of the pipe. A similar procedure is followed using equations (13) and (15), thereby finding the circumferential and axial stresses throughout the thickness of the pipe.

It was noted earlier that the radial displacement, remote from the pipe junction, used in determining the total deflection would be an average value of the radial displacements found at the inner and outer surfaces of the pipe, i.e.,

$$u = \frac{1}{2} [u(a) + u(b)] \qquad (37$$

From this it can easily be shown that

$$u = \left(\frac{a+b}{2}\right) [-\mu\varepsilon_z + \frac{2(1+\mu)}{b^2 - a^2} \int_a^b \alpha Trdr] \qquad (38)$$

where ε_z is found from equation (14). Thus, this radial displacement can be determined for each pipe by evaluating the integral of the polynomial for the temperature distribution at the inner and outer surfaces of the pipe.

The next step is to find the bending moment, M_o and the radial force V_o at the junction of the two pipes. This procedure was outlined briefly in the preceding section, but will be shown in some detail here. Substituting equation (20) and the values obtained for u and u_1, from equation (38), into equation (18) thus

$$\frac{1}{2\beta^3 D}\,[\beta M_o' + V_o] + u = \frac{1}{2\beta_1^3 D_1}\,[\beta_1 M_{o1}' + V_o] + u_1$$

where as before, subscript 1 is used to distinguish between the two pipes. Now substituting $M_o' = M_o + M_\infty$ and $M_{o1}' = M_o + M_{\infty 1}$, and rearranging

$$\left[\frac{1}{2\beta^2 D} - \frac{1}{2\beta_1^2 D_1}\right] M_o + \left[\frac{1}{2\beta^3 D} - \frac{1}{2\beta_1^3 D_1}\right] V_o = u_1 - u - \left[\frac{M_\infty}{2\beta^2 D} - \frac{M_{\infty 1}}{2\beta_1^2 D_1}\right] \qquad (39)$$

Substituting equations (22) into equation (18a)

$$\frac{1}{2\beta^2 D}\,[2\beta M_o' + V_o] = \frac{1}{2\beta_1^2 D_1}\,[2\beta_1 M_{o1}' + V_o]$$

then substituting as above and rearranging

$$\left[\frac{1}{\beta D} + \frac{1}{\beta_1 D_1}\right] M_o + \left[\frac{1}{2\beta^2 D} + \frac{1}{2\beta_1^2 D_1}\right] V_o = -\left[\frac{M_\infty}{\beta D} + \frac{M_{\infty 1}}{\beta_1 D_1}\right] \qquad (40)$$

Equations (39) and (40) are easily solved for the mechanical reactions M_o and V_o after which the appropriate dislocation reaction moments

$$M_o' = M_o + M_\infty$$

$$M_{o1}' = M_o + M_{\infty 1}$$

may be determined.

Now that the dislocation reactions, M_o', M_{o1}', and V_o, have been determined, the stresses which they produce in each pipe can be computed. These stresses may then be superposed on the thermal stresses already obtained and shown in equations (12), (13) and (15), obtaining the total stresses as defined in equation (26). Next computing the shearing stress, equation (27), and the principal stresses, equation (29), the significant stress may be determined. It is worth repeating here that this significant stress is the largest difference between any pair of principal stresses. Hence, it is a function of the radial coordinate, as well as the axial coordinate of the pipe. Also it may be useful to know which pair of principal stresses produce this maximum.

The computer program takes the absolute value of the difference of each pair of the three principal stresses, and then selects the largest of these as the maximum significant stress. This is done at increments of the radius throughout the thickness of each pipe. It has been noted earlier that the radial stress is zero at the inner and outer surfaces of the pipe. Also the circumferential and axial stresses remote from the pipe junction are maximum at either the inner or outer surface for a large class of fluid temperature variations. Therefore, it seems reasonable that in taking differences, the largest difference might be at either inner or outer surface. In fact, in certain important examples of specified time-temperature relationships (i.e. sudden change in internal temperature), it has been observed empirically by the writer that this maximum does occur at either the inner or outer surface, but admittedly this may not always be the case. In any case, finding the radial position at which the significant stress is maximum is simplified by merely finding the significant stress at given increments of radius throughout the pipe thickness.

The axial position at which the significant stress is maximum is that position at which the dislocation stresses are maximum. As noted earlier, these stresses are maximum at the point of maximum bending moment which may occur at the junction ($z = 0$) or at a mathematical extremum of the bending moment. Equation (24) gives the moment as a function of z, β, M_o' and V_o. But now β, M_o' and V_o are known. Therefore, taking the derivative of the moment with respect to z and setting it equal to zero, the points of extremal moment can be determined.

$$M(z) = \frac{V_o}{\beta} [e^{-\beta z} \sin\beta z] + M_o' [e^{-\beta z}(\cos \beta z + \sin\beta z)] \qquad (24)$$

Setting the derivative equal to zero,

$$0 = V_o [e^{-\beta z}(\cos \beta z - \sin \beta z)] - \beta M_o' [e^{-\beta z}(2 \sin \beta z)]$$

Rearranging to a form readily usable on the computer, and solving for z

$$z = \frac{1}{\beta} \text{Arctan} \left[\frac{V_o}{2\beta M_o' + V_o}\right] \qquad (41)$$

This method is employed in the computer program. In this manner, the first (and only physically meaningful) point of extremal moment can be found for each pipe, being sure to use the appropriate value of β. It must be remembered here, that the value of the dislocation moment at the pipe junction, M_o', has already been determined. In examining certain important temperature-time histories, such as those corresponding to sudden or rapid changes in internal fluid temperature, it has been observed empirically that the maximum bending moment for each pipe occurs at the pipe junction. From these examples, it was noted that the magnitude of the moment, found by substituting the value of z obtained from equation (41) into equation (24), was always less than the magnitude of the moment at the pipe junction. In fact, in all the examples studied in this thesis investigation, not only is this moment at the junction a maximum, but the moment decreases rapidly away from the junction, oscillating above and below a value of zero. The program uses either the end moment or the moment at the axial position given by equation (41), whichever is larger, in computing the dislocation stresses.

If the temperature distribution were such that the maximum significant stress did not occur at the inner or outer surfaces, the user of program PAM1 still has the option of wholesale determinations at different radii from which a maximum may be found. If the maximum significant stress did not occur at the end ($z = 0$), the preceding determination, which is based upon maximizing M without considering η or V would probably not give the axial position of greatest significant stress. Because the cases which seemed to be of greatest interest lead to maximum significant stress occuring at either the inner or outer surface, the program PAM1 in its present form does not include the capability of making determinations at the axial positions other than for ($z = 0$) and the value of z given in equation (41). Accordingly, for cases in which the maximum significant stress occurs between the inner and outer radii, the program does not presently provide a capability of finding the critical spot and determining the true maximum significant stress. However, there should be no great difficulty in modifying the program to provide this capability if desired.

Once the maximum significant stress is found for a given time, the same procedure can be followed for the next time increment. It may be worth repeating that two values of maximum significant stress are determined at each given time epoch-one for each pipe. The temperature distribution is known for that time increment, therefore, a polynomial can be found to represent the temperature, and then that polynomial can be used to find the next significant stress. This method is employed in the computer program making it possible to find a maximum significant stress over any time period.

IV. DIGITAL COMPUTER PROGRAM

This section will serve as a summary for all that has preceded it in the sense that now all the theoretical techniques and the methods devised to carry out these techniques have been incorporated into the program PAM1 which is immediately usable for the solution of the particular problem at hand. This program is written in Fortran IV language and employs only normal Fortran functions.

Appendices A and B will give a complete listing of the program, as well as instructions for its use. The writer's purpose in this section is to outline the program (a flow chart in words perhaps) thereby describing the various sub-programs employed and woven together to form the final program

As indicated in the preceding section, the whole idea of applying the theoretical analysis devised herein to a transient-state problem rests on the premise that a transient temperature distribution could be obtained, and it was shown that by using a numerical method, such a distribution can be obtained. In view of the fact that this temperature had to be integrated in order to find the stresses which it produces, the temperatures and the corresponding radii found from BETTY1, were used as data points; then a polynomial was fitted to these points. The program employs the least squares method and Gauss elimination to find the coefficients of the polynomials.

PAM1 reads data for one pipe and solves for the coefficients of the polynomial for every instant in time at which a temperature distribution has been determined. This continues until the problem-end-time[2] has been reached, with the program storing each of the coefficients thus far obtained. Next, the program reads data for the joining pipe and solves for the coefficients of a similar polynomial. Now, however, at each time interval the program recalls the coefficients for the other pipe at the corresponding time. The program continues to do this until the computation terminates. For each time increment there is a polynomial for the temperature distribution for each pipe; hence, the stresses can be determined for every epoch of time.

As shown in Section II, these stresses are determined by integrating the polynomial, which is a function of radius, multiplied by the radius (i.e. $\int Trdr$). This process is carried out in subroutine PINT. Subroutine PVAL is then used to evaluate the integral at any given radius, as well as at the inner and outer surfaces of the pipe. In calculating the circumferential and axial stresses, the temperature at a given radius is required. Subrouting TVAL evaluates only the polynomial for the temperature distribution in order to obtain the temperature at a given radius.

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[2]This is an upper limit, introduced as an input to PAM1. The computation terminates when time has been incremented to the value of problem-end-time.

The same techniques are employed in finding the radial displacement remote from the pipe junction. Subroutines PINT1, PVAL1, and TVAL1 perform the same procedures in finding the stresses and radial displacement for the adjoining pipe.

The program then solves equations (39) and (40) for the unknowns M_o and V_o, the total bending moment and radial force at the pipe junction, and the corresponding dislocation reactions (M'_o, V_o) and (M'_{o1}, V_o) are next determined. At this point, equation (41) is used to determine the point of maximum bending moment. The moment at this point is then computed and compared to the moment at the pipe junction, and the one of greater magnitude is used in determining the dislocation stresses. The total stresses are then determined and the maximum significant stress as defined in Section II is obtained.

Each of the stresses obtained thus far depends upon the radius, so that the program can find these stresses for each pipe at any radius throughout the thickness of the pipe. Since it has been this writer's experience that the maximum significant stress occurs at either the inner or outer surface, the program offers the option of either finding only those stresses at the inner or outer radii or all stresses at given increments of radius. This depends on the wish of the user and is elected through the use of a data card.

In any case, once these stresses are determined for a given time the program continues to the next instant in time, calculates the coefficients of the polynomial for the second pipe, recalls the coefficients already determined for the first pipe, and performs all the evaluations described above.

These procedures continue until the problem-end-time is reached. After determining the significant stress for each epoch of time, the program calculates the largest of these, thereby determining the maximum significant stress encountered over the problem time for each pipe. PAM1 also furnishes the time and location of this maximum stress, as well as indicating the nature of the two principal stresses responsible for it. It should be repeated that the output includes two such combinations of results - one for each pipe.

APPENDIX A

Instructions for use of Program PAM1

This program is designed for the IBM series 360 computer; however, there is very little machine dependence. Testing and execution thus far has been on an 360/67 configuration. All quantities beginning with the alphabetic letters I, J, L, M, and N are integer variables, used for distinguishing specific locations in the pipe wall, used as counters for storing purposes, and used as limits for DO-loops. All other quantities are continuous (REAL *8, i.e., double precision) variables. Many of these variables differ in appearance only by the number (1) placed at the end of the variable. This is to distinguish related variables for each of the joining pipes. For instance, ELAS is used to denote the modulus of elasticity for one pipe, while ELAS1 denotes the modulus of elasticity for the joining pipe.

The program determines temperatures at the centers of each of the N layers into which the pipe wall is divided. The arrangement of these layers is one half-layer, (N-1) full layers, and a final half-layer. An example for N=5 layers is illustrated in Figure A-1.

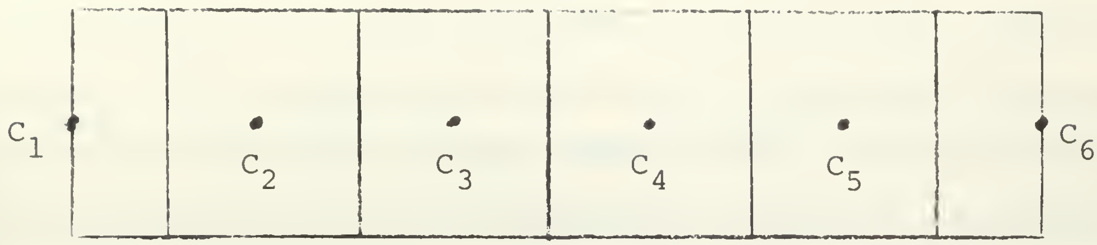

Figure A-1

Note that the centers are numbered from C_1 to C_{N+1}. Therefore, in the program the given value of N is increased by unity so as to permit numbering the centers as illustrated in the figure above. One important consequence of this is that in fitting the polynomial to the temperature distribution, there are N+1 temperatures and N+1 radii used as data points. The number of layers should be between four and twenty, inclusive, but if greater than ten, it should be an even number. Otherwise the program automatically changes N to conform to these conditions.

The temperatures, coefficients of the polynomials, and related stresses are calculated at discrete epochs, uniformly separated by time-interval DT. The given value of DT, as selected by the user is reduced if necessary to assure convergence of the computation. As this calculated value may be different for each of the connected pipes, care must be taken to see that the smaller value is used. Because of the sequence of operations involved, it may happen that temperature evaluations have been completed for one pipe before analysis of the parameters relating to the second pipe has revealed that a shorter time interval should have been used. In this case an appropriate message is printed and the computation ceases; it is then necessary for the user to make input modifications as advised and to restart the program. However, if this case does not arise, the computation procedes to completion as desired. Another method of insuring that the same value of DT is used for both pipes is to remove temporarily the card DT = BB which precedes statement 31. This enforces the input value of DT. However, if this is done, one should be alert to the possibility of instability or non-convergence in the calculation.

The data deck is arranged in two parts. The first part contains material and geometrical properties which are constant for each pipe. This part consists of two data cards. The second part of the data deck contains the numbers of layers into which the pipe is divided, time interval, problem-end-time, data for those properties of the material and fluid which may vary with temperature, as well as information regarding the internal fluid time-temperature relationships and initial pipe temperature. The degree of the polynomial desired should also be selected at this time. Although there has not been sufficient experimental use of the program to warrant making definite statements, tentatively it may be recommended that the degree of the polynomial should not exceed 0.6 N (where N is the number of layers). Unless the fluid temperature fluctuates with time there seems to be no reason to employ a polynomial of higher than fourth degree. Another important input is the option parameter NSTAR which determines if the stresses will be calculated at only the inner and outer surfaces or at given increments of radius throughout the wall of the pipe. Presently evaluations are made for $r = a + n(b - a)/100$; $n=0, 1, \ldots, 100$. However the interval may be modified by changing cards 900 and 901.

Detailed instructions for the preparation of each data card follow.

Part I

Card 1

Column	Format	Description or Value
1-10	F 10.0	Young's modulus of elasticity, psi
11-20	F 10.0	coefficient of linear thermal expansion, $°F^{-1}$
21-30	F 10.0	Poisson's ratio
31-40	F 10.0	inner radius of pipe, inches
41-50	F 10.0	outer radius of pipe, inches

Card 2

This card is punched exactly as Card 1 using parameters now for the connecting pipe.

In the following, note that all integer inputs must be right-justified in their respective fields.

Part II Card 3

Column	Format	Description or Value
1-3	I3	degree of polynomial desired
4-6	I3	1 or 2. Number one (1) is used to denote first pipe; number two (2) is used to denote second pipe.
7-9	I3	1 or 2. Option parameter one (1) used to calculate stresses at inner and outer surface only, while option parameter two (2) calculates stresses at increments of radius throughout pipe wall.

Card 4

Column	Format	Description or Value
1	I1	1. (A digit in this position, cards 5 through 11 merely sets up appropriate internal branching)
2-3	I2	number of layers into which pipe thickness is divided
4-5	I2	1. Number one (1) must be punched here. Program BETTY1 had capabilities to print output only on certain time intervals. Program PAM1 does not.
6-10	I5	blank
11-20	F10.4	outside diameter of pipe, inches
21-30	F10.4	pipe thickness, inches
31-40	F10.4	time interval, seconds
41-50	F10.4	maximum problem time, seconds

Card 5

Column	Format	Description or Value
1	I1	2
11-20	F10.4	surface heat transfer coefficient, Btu/hr - sq.ft - °F
21-30 31-40 41-50	F10.4	Higher coefficients which give variation with respect to temperature according to the formula; Property = F5 + X*(F6 + X*(F7 + X*F8)) where X denotes temperature and F5 through F8 denote the four entries in columns 11 through 50 respectively. If the higher coefficients are left blank, a value of zero is used.

Card 6

1	I1	3
11-20	F10.4	thermal conductivity, Btu/hr - ft - °F
21-30 31-40 41-50	F10.4 F10.4 F10.4	higher coefficients used as in Card 5

Card 7

1	I1	4
11-20	F10.4	specific heat, Btu/lb - °F
21-30 31-40 41-50	F10.4 F10.4 F10.4	higher coefficients used as in Card 5

Card 8

Column	Format	Description or Value
1	I1	5
11-20	F10.4	density, lbs/cu.ft.
21-30 31-40 41-50	F10.4 F10.4 F10.4	higher coefficients used as in Card 5

Card 9

1	I1	6
11-20	F10.4	time, seconds, at which the internal fluid temperature is that entered in columns 21-30
21-30	F10.4	temperature of internal fluid, °F. Type 6 cards govern the time-temperature relationship of the internal fluid and as many as thirty may be used. These can be denoted as Cards 9b, 9c, etc.

Card 10

1	I1	7
11-20	F10.4	Initial uniform temperature of pipe, °F

Card 11

1	I1	8 Number eight (8) signals computer to begin computation

Part II of the data deck must now be repeated using the same formats but this time using those parameters for the connecting pipe (i.e., the pipe identified in Card 2). These cards follow directly after the card with the number eight (8) punched in column 1, and they must also end with a card with 8 punched in column 1.

APPENDIX B

Glossary of Symbols Used in Program

Listing of program starts on page 59

A(I)	cross sectional area at center number I
A3	inner radius of pipe
AA,BB3,RR	quantities useful in calculating $\int Trdr$
AAA	quantity useful in calculating SIGMAR
AENDX	absolute value of ENDX
AENDMO	absolute value of ENDMO
AL(I)	K(X(I))/(SPHT(X(I))*DENS(X(I))), a useful combination
ARG	value for which integral is evaluated
ALPHA	coefficient of linear thermal expansion
AYE	input from data card
B	input from data card
B3	outer radius of pipe
BBB3	quantity useful in calculating SIGMAR
BBX	value of BETA times z
BEST	maximum significant stress for a given time
BETA	$\sqrt[4]{\dfrac{3(1-\mu^2)}{r^2h^2}}$
BIG	largest value of RT, TZ and ZR
BUZZ(I)	variable used to store radius of maximum significant stress for each time interval

BX	BBX
C	input from data card
C3	value of $\int Trdr$ from inner radius to some radius, R
CCC	quantity useful in calculating SIGMAT
D	input from data card
D3	value of $\int Trdr$ from inner radius to outer radius
DENOM	a quantity useful in calculating TAVG
DENS(Z)	metal density at temperature z
DENA, DENB, DENC, DEND	coefficients for determining DENS(Z)
DEL	plate flexural rigidity = $Eh^3/12(1 - \mu^2)$
DELTA1	ΔT_1 for USAS B31.7 Nuclear Power Piping Code
DELTA2	ΔT_2 for USAS B31.7 Nuclear Power Piping Code
DETT1	absolute value of DELTA1
DETT2	absolute value of DELTA2
DONE, DTWO	quantities used in interpolating in subroutine FLUTEM
DD	quantity useful in determining radial displacement
DDD	quantity useful in obtaining SIGMAZ
DR	thickness of a layer
DT	time interval
E	input from data card
ELAS	modulus of elasticity
EMINF	bending moment remote from pipe junction
ENDX	bending moment as a function of z
ENDP	dislocation bending moment

ENDMO	total bending moment at pipe junction
EPSZ	unit elongation in axial direction
F	input from data card
F3	quantity useful in calculating SIGMAR
FELTA1	cumulatively largest value of DETT1
FELTA2	cumulatively largest value of DETT2
FLUI(I), FLUO(I)	interior, exterior fluid temperature[3]
FLUTEM (Z,A,B)	subroutine entered with time Z and giving A,B as corresponding inside, outside fluid temperatures
FRAC	numerical constant used in making initial estimate of inner surface temperature
FLU	interior fluid temperature
G	quantity useful in calculating SIGMAT
GAKA	radius of maximum significant stress
GAUSS2	subroutine used for finding coefficients of polynomial for temperature distribution
GLUG1	value of DETT1 at time TT1
GLUG2	value of DETT2 at time TT2
H	interior surface film coefficient
H3	quantity useful in calculating SIGMAZ
HO	external surface film coefficient (taken as zero corresponding to perfect insulation)
IDIMT	number of coefficients in temperature polynomial
IDIMX	number of coefficients in an associated temperature polynomial

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[3]Reference to exterior fluid temperature is for a planned future capability of BETTY1; no such quantity is actually used in the calculations herein described.

IDIMY	IDIMX + 1
JAP	counter used in storing X3(I)
JUAY	integer parameter for preparing headings
K(Z)	metal thermal conductivity at temperature Z
KA,KB,KC,KD	coefficients used in determining K(Z)
KAY(I)	K(X(I))*A(I), useful combination
LORDER	degree of polynomial desired
LL	integer parameter for preparing headings
MO	LORDER + 1
MOM	counter used in storing significant stress for each time interval
MOP	counter used in storing significant stress of adjoining pipe for each time interval
N_1, N_2, N_3, N_4	input from data card
NO	number of points used to make up polynomial for temperature distribution
NCOUNT	1 (in the present application of BETTY1)
NFLAG	used to distinguish two joining pipes
NPRINT	number of time intervals between lines of printed output
NSTAR	option parameter in data deck
OD	outside diameter of pipe
P	a useful parameter which takes care of conversion units
P3	quantity useful in obtaining end moment and radial force
POIS	Poisson's ratio
PINT	subroutine used to integrate polynomial
PVAL	subroutine used to evaluate $\int Trdr$

Q	quantity useful in calculating end moment and radial shear force
R	radial coordinate of pipe
RAD(I)	variable used to store radius of significant stress
ROCK	radius of maximum significant stress for a given time
RT	absolute value of ST1 - ST2
RY	radial distance measured from mid-surface of pipe
S3	quantity useful in calculating end moment and radial force
SHEAR	radial force as a function of axial coordinate
SIG(I)	variable used to store maximum significant stress for each time interval
SIGMAR	radial stress remote from pipe junction
SIGMAT	circumferential stress remote from pipe junction
SIGMAZ	axial stress remote from junction
SIGTX	circumferential stress due to bending moment and radial force
SIGZX	axial stress due to bending moment and radial force
SIGR	total radial stress
SIGT	total circumferential stress
SIGZ	total axial stress
SPHT(Z)	metal specific heat at temperature Z
SPHA, SPHB, SPHC, SPHD	coefficients for calculating SPHT(Z)
SSIG	maximum significant stress encountered
ST1	principal stress 1

ST2	principal stress 2
ST3	principal stress 3
STRESS(I)	variable used to store significant stress for a given radius
SURFCO(Z)	inner surface film coefficient at temperature Z
SURA, SURB, SURC, SYRD	coefficients for calculating SURFCO(Z)
T	problem-time
TAU	shearing stress
TIMA	time at which maximum significant stress in encountered
TIME(I)	variable used to store time of maximum significant stress for each time interval
TMAX	problem-end-time
TOM	value of DT for first pipe
TT1	time for which value of FELTA1 is recorded
TT2	time for which value of FELTA2 is recorded
TT3	temperature as obtained from polynomial for temperature distribution
TVAL	subroutine used to evaluate TT3
U	radial displacement remote from pipe junction
UAY(I)	alphabetic data for headings
UU	value of right hand side of equation (39)
UUU	value of right hand side of equation (40)
V	deflection at pipe junction
VO	radial force at pipe junction
W	quantity useful in calculating end moment and radial force

WT	wall thickness
X(I)	temperature at center I
XAVG	average temperature throughout thickness of pipe
XB	value of z where bending moment is extremal
X2(I)	data point denoting radius used in fitting a polynomial
X3(I)	coefficient for polynomial for temperature distribution
XFO	outside fluid temperature
XX(I)	newly calculated value of X(I)
Y(I)	coefficients of polynomial for temperature after integration
Z	a useful quantity for calculating XAVG
ZED	used in FLUTEM
ZR	absolute value of ST3 - ST1

Recall that there are also a number of other variables with names similar to those above but with a terminal 1.

PROGRAM PAM1

```
      IMPLICIT REAL*8 (A-H,K,O-Z), INTEGER (I,J,L,M,N)
      DIMENSION A(20),X(20),XX(20),KAY(20),AL(20),TEM(30),FLUI(30),
     1FLUO(30),UAY(32),SIGMA(10,30),B2(10,20),A2(10,20),X2(10),X12(50),
     2Y12(50),ITITLE(12),Z12(50),Z2(50),X22(10,20),A22(10,30),X3(100),
     3X31(100),X33(100,100),Y(100),Y1(100),XX3(100),XX31(100)
     4,STRESS(30),STRES1(30),SIG(100),TIME(100),SIG1(100)
     5,RAD(30),RAD1(30),BUZZ(100),BUZZ1(100)
      DATA UAY     /8H    TIME,8H   DELTA,8H   DELTA,8H     AVG.,8H   FLU
     1ID,8H   INNER,8H   OUTER,8H SURFACE,8H SECONDS,8H      ONE,8H     T
     2WO,8H   TEMP.,8H   TEMP.,8H      ONE,8H      TWO,8H    THREE,8H    FO
     3UR,8H    FIVE,8H     SIX,8H   SEVEN,8H    EIGHT,8H     NINE,8H     T
     4EN,8H        ,8H  TWELVE,8H        ,8H       14,8H         ,8H
     516,8H        ,8H      18,8H  CENTER/
      COMMON TEM,FLUI,FLUO,A2,MO
      SURFCC(Z)=((SURD*Z+SURC)*Z+SURB)*Z+SURA
      SPHT(Z)=((SPHD*Z+SPHC)*Z+SPHB)*Z+SPHA
      DENS(Z)=((DEND*Z+DENC)*Z+DENB)*Z+DENA
      K(Z)=((KD*Z+KC)*Z+KB)*Z+KA
      READ(5,71) ELAS,ALPHA,POIS,A3,B3
      READ(5,71) ELAS1,ALPHA1,POIS1,A31,B31
   71 FORMAT(5F10.0)
  280 CONTINUE
      XAVG=0.
      I =0
      II=0
      DO 290 J=2,20
  290 X(J)=0.
      XFO=0.
   10 READ(5,11) LORDER,NFLAG,NSTAR
   11 FORMAT(3I3)
  300 READ(5,820) N1, N2, N3, N4, AYE, B, C, D, E, F
  820 FORMAT(I1,2I2,I5,6F10.4)
      IF(NFLAG.NE.1) GO TO 506
      IDIMX=LORDER+2
      GO TO 507
  506 IDIMX1=LORDER+2
      IDIMY1=IDIMX1+1
      IDIMT1=IDIMX1-1
  507 IDIMY=IDIMX+1
      IDIMT=IDIMX-1
      IF(N1.EQ.1) GO TO 310
      IF(N1.EQ.2) GO TO 320
```

```
      IF(N1.EQ.3) GO TO 330
      IF(N1.EQ.4) GO TO 340
      IF(N1.EQ.5) GO TO 350
      IF(N1.EQ.6) GO TO 360
      IF(N1.EQ.7) GO TO 370
      IF(N1.EQ.8) GO TO 380
  310 N=N2
      NPRINT=N3
      OD=AYE
      WT=B
      DT=C
      TMAX=D
      GO TO 300
  320 SURA=AYE
      SURB=B
      SURC=C
      SURD=D
      GO TO 300
  330 KA=AYE
      KB=B
      KC=C
      KD=D
      GO TO 300
  340 SPHA=AYE
      SPHB=B
      SPHC=C
      SPHD=D
      GO TO 300
  350 DENA=AYE
      DENB=B
      DENC=C
      DEND=D
      GO TO 300
  360 I=I+1
      II=II+1
      IF(I.GT.30) I=I-30
      TEM(I)=AYE
      FLUI(I)=B
      FLUO(I)=C
      GO TO 300
  370 X(1)=AYE
      GO TO 300
  380 CONTINUE
      T=0.
      NCOUNT=0
      PRINT 830
  830 FORMAT('1',5X,'THERMAL TRANSIENTS PROBLEM FOR USAS B31.7 CODE.')
      IF(OD.EQ.0.) PRINT 831, WT
```

```
      IF(OD.NE.0.) PRINT 832, OD, WT
  831 FORMAT(/,5X,'FLAT SLAB OF ',F8.4,' INCH THICKNESS.')
  832 FORMAT(/,5X,'TUBE OR PIPE OF ',F8.4,' INCH CUTSIDE CIAMETER AND',
     1/,5X,'OF ',F8.4,' INCH WALL THICKNESS.')
      PRINT 833, CT, TMAX
  833 FORMAT(/,5X,'TIME INTERVAL = ',F8.5,' SECCNDS:',/,5X,'MAXIMUM',
     1' TIME = ',F8.2,' SECCNDS.')
      PRINT 834, SURA, SURB, SURC, SURD
  834 FORMAT(/,5X,'SURFACE COEFFICIENT OF HEAT TRANSFER EQUALS ',
     1F8.3,' BTU/HR-FTSQ-DEGF;',/,5X,'HIGHER COEFFICIENTS ARE ',
     2F8.3,', ',F8.3,', AND ',F8.3,'.')
      PRINT 835, KA, KB, KC, KD
  835 FORMAT(/,5X,'THERMAL CCNDUCTIVITY = ',F8.4,' BTU/HR-FT-DEGFF',
     3/,5X,'HIGHER COEFFICIENTS ARE ',F8.5,', ',F8.5,', AND ',F8.5,'.')
      PRINT 836, SPHA, SPHB, SPHC, SPHD
  836 FORMAT(/,5X,'SPECIFIC HEAT = ',F8.5,' BTU/LB-DEGF;',
     3/,5X,'HIGHER COEFFICIENTS ARE ',F8.5,', ',F8.5,', AND ',F8.5,'.')
      PRINT 837, DENA, DENB, DENC, DEND
  837 FORMAT(/,5X,'DENSITY = ',F8.3,' POUNDS PER CUBIC FOOT;',
     3/,5X,'HIGHER COEFFICIENTS ARE ',F8.3,', ',F8.3,', AND ',F8.3,'.')
      IF(NFLAG.NE.1) GO TO 850
      PRINT 840, ELAS
  840 FORMAT(/,5X,'YOUNGS MODULUS OF ELASTICITY =',F10.0,
     1'POUNDS PER SQUARE INCH.')
      PRINT 841, ALPHA
  841 FORMAT(/,5X,'COEFFICIENT OF LINEAR THERMAL EXPANSION=',F10.7,
     1' PER DEGF.')
      PRINT 842, POIS
  842 FORMAT(/,5X,'PCISSCNS RATIO =',F8.3,'.')
      GO TO 843
  850 PRINT 840, ELAS1
      PRINT 841, ALPHA1
      PRINT 842, POIS1
  843 PRINT 838, N,NPRINT
  838 FORMAT(/,5X,'DESIGNATED NUMBER OF LAYERS IS ',I2,'.',/,5X,
     1'PRINTING TAKES PLACE EVERY ',I2,'TH TIME INTERVAL.')
      IF(NFLAG.NE.1) PRINT 95
   95 FORMAT(/,5X,'FOR DETERMINING TYPE OF SIGNIFICANT STRESS THE FOLLOW
     1ING IS USED:',/,5X,'TYPE 1 DENCTES DIFFERENCE BETWEEN STRESS ONE A
     2ND STRESS TWO.',/,5X,'TYPE 2 DENOTES DIFFERENCE BETWEEN STRESS TWO
     3 AND STRESS THREE.',/,5X,'TYPE 3 DENOTES DIFFERENCE BETWEEN STRESS
     4 THREE AND STRESS ONE.')
      WT=WT/12.
      IF(II.GT.1) GO TO 839
      FLUI(2)=FLUI(1)
      FLUO(2)=FLUO(1)
      TEM(2)=TMAX
      II=2
```

```
      I=2
  839 CONTINUE
      IF(II.GE.30) NOPT=30
      IF(II.LT.30) NOPT=II
      NOPM=NOPT-1
      DO 204 I=1,NOPM
      IP=I+1
      DO 204 J=IP,NOPT
      IF(TEM(I).NE.TEM(J)) GO TO 204
      IF(FLUI(I).EQ.FLUI(J).AND.FLUO(I).EQ.FLUO(J)) GO TO 203
      PRINT 8000
      PRINT 821
  821 FORMAT(5X,'ONE OR MORE TEMPERATURE INPUTS ARE NOT CONSISTENT,',/,
     15X,'LAST INPUT ACCEPTED AS CORRECT.')
  203 TEM(I)=1.D+75
  204 CONTINUE
      DO 205 I=1,NOPM
      IP=I+1
      DO 205 J=IP,NOPT
      IF(TEM(I).LT.TEM(J)) GO TO 205
      TEMPA=TEM(I)
      TEMPB=FLUI(I)
      TEMPC=FLUO(I)
      TEM(I)=TEM(J)
      FLUI(I)=FLUI(J)
      FLUO(I)=FLUO(J)
      TEM(J)=TEMPA
      FLUI(J)=TEMPB
      FLUO(J)=TEMPC
  205 CONTINUE
      NOP=0
      DO 105 I=1,NOPT
      Z=TEM(I)
      IF(Z.LT.1.D+75) NOP=NOP+1
  105 CONTINUE
      NOPT=NOP
      PRINT 107
      DO 106 I=1,NOPT
  106 PRINT 108,TEM(I),FLUI(I),FLUO(I)
  107 FORMAT(/,5X,'AMBIENT TEMPERATURES ARE:',//,9X,'TIME  INSIDE ',
     1'OUTSIDE')
  108 FORMAT(5X,F8.1,2F8.2)
      PRINT 8000
 8000 FORMAT(/)
      IF(N.GT.20) PRINT 8002, N
 8002 FORMAT(5X,'INPUT ASKS FOR ',I4,' LAYERS.  THIS HAS BEEN',/,
     15X,'REPLACED BY TWENTY LAYERS, A PROGRAM MAXIMUM.')
      IF(N.LT.4) PRINT 8003, N
```

```
8003 FORMAT(5X,'INPUT ASKS FOR ',I4,' LAYERS.  THIS HAS BEEN',/,
    15X,'REPLACED BY FOUR LAYERS, A PROGRAM MINIMUM.')
     IF(N.GT.20) N=20
     IF(N.LT.4) N=4
     NN=1
     IF(N.LE.10) GO TO 20
     NN=N/2
     NP=2*(N-NN)
     NN=2
     PRINT 8000
     PRINT 8004, N, NP
8004 FORMAT(5X,'INPUT ASKS FOR ',I4,' LAYERS.  THIS HAS BEEN',/,
    15X,'REPLACED BY ',I4,' LAYERS, AN EVEN NUMBER.')
     N=NP
  20 CONTINUE
     N=N+1
     NO=N
     DO 21 I=2,N
     IF(X(I).EQ.0.) X(I)=X(1)
  21 CONTINUE
     EN=N
     NM=N-1
     ENM=NM
     DR=WT/ENM
     DO 22 I=1,N
     EJ=N-I
     IF(OD.LE.0.) A(I)=1.
     IF(OD.GT.0.) A(I)=(.5*OD-EJ*DR)/(.5*OD-ENM*DR)
  22 CONTINUE
     CALL FLUTEM(T,XF,ZED)
     H=SURFCO(.5*(XF+X(1)))
     B=H*DR/K(X(1))
     BB=A(N)/A(1)
     BBB=B+BB
     B=DR**2*SPHT(X(1))*DENS(X(1))/(3.*BBB*K(X(1)))
     BB=B*3600.
     J=0
  23 IF(BB.LT.10.) GO TO 24
     J=J+1
     BB=BB/10.
     GO TO 23
  24 IF(J.LE.0) GO TO 25
     I=BB
     BB=1.
     IF(I.GT.2) BB=2.
     IF(I.GT.5) BB=5.
     BB=BB*10.**J
     GO TO 27
```

```
 25 IF(BB.GT.1.) GO TO 26
    J=J+1
    BB=BB*10.
    GO TO 25
 26 I=BB
    BB=1.
    IF(I.GT.2) BB=2.
    IF(I.GT.5) BB=5.
    BB=BB/10.**J
 27 CONTINUE
    PRINT 8000
    IF(DT.GT.BB.CR.DT.LE.0.) GO TO 28
    PRINT 29, DT, BB
    GO TC 31
 28 IF(DABS(DT-BB).LT..001*BB) PRINT 301, DT
    IF(DABS(DT-BB).GE..001*BB) PRINT 30, DT, BB
301 FORMAT(5X,'GIVEN VALUE CF DT, 'D12.3,' SECONDS, WILL BE USED.')
 29 FORMAT(5X,'GIVEN VALUE OF DT WAS ',D12.3,' SECONDS. THIS WILL',/,
   15X,'BE USED EVEN THOUGH A LARGER VALUE, ',D12.3,' SECONDS,',/,
   25X,'CCULD HAVE ASSUREC CONVERGENCE.')
 30 FORMAT(5X,'GIVEN VALUE, DT EQUALS ',D12.3,' SECONDS, REPLACED',/,
   15X,'BY DT ECUALS ',D12.3,' SECONDS TO ASSURE CONVERGENCE.')
    DT=BB
    IF(NFLAG.NE.1) GO TO 508
    TOM=DT
    GC TC 31
508 IF(DT.GE.TOM) GO TO 509
    IF(DT.LT.TCM) GC TO 510
509 DT=TCM
    GO TO 31
510 PRINT 511
511 FORMAT(/,5X,'VALUE CF DT CALCULATED FOR PIPE TWC IS LESS THAN THAT
   1 CALCULATED FOR PIPE ONE. RESUBMIT PROGRAM USING THE SMALLER VALUE
   2 CF DT FOR BCTH PIPES.')
    GO TO 5000
 31 P=DT/(360C.*DR**2)
    DELTA1=0.
    DELTA2=0.
    FELTA1=0.
    FELTA2=0.
    GLUG1=0.
    GLUG2=0.
    TT1=0.
    TT2=0.
    DNCM=-.5*(A(1)+A(N))
    DO 32 I=1,N
 32 DNOM=DNCM+A(I)
    JUAY=13+NN
```

```
      LL=12+NM
      PRINT 403
  403 FORMAT(///)
      WRITE(6,40)(UAY(I),I=1,6),(UAY(32),J=JUAY,LL,NN),UAY(7)
      WRITE(6,40)(UAY(I),I=9,13),UAY(8),(UAY(J),J=JUAY,LL,NN),UAY(8)
   40 FORMAT(2X,16A8)
      CALL FLUTEM(T,FLU,XFO)
      PRINT 41,T,DELTA1,DELTA2,XAVG,FLU,(X(I),I=1,N,NN)
   41 FORMAT(2X,16F8.2)
      JAP=1
      NMY=N-1
   44 IF(NFLAG.NE.1) GO TO 400
      X2(1)=A3
      DO 499 I=1,NMY
      X2(I+1)=X2(I)+(B3-A3)/(N-1)
  499 CONTINUE
      GO TO 498
  400 X2(1)=A31
      DO 497 I=1,NMY
      X2(I+1)=X2(I)+(B31-A31)/(N-1)
  497 CONTINUE
  498 R=A3
      R1=A31
      RY=-((B3-A3)/2)
      RY1=-((B31-A31)/2)
      MO=LCRDER+1
      EP=.0000001
      DO 13 IK=1,MO
      DO 13 L=1,MC
      ME=IK+L-2
      SIGMA(IK,L)=0.0
      DO 13 M=1,NC
      IF(ME-0) 19,19,38
   38 SIGMA(IK,L)=SIGMA(IK,L)+X2(M)**ME
      GO TO 13
   19 SIGMA(IK,L)=SIGMA(IK,L)+1.
   13 CONTINUE
      DO 14 I=1,MC
      LA=I-1
      B2(I,1)=0
      DO 14 NA=1,NO
      IF(LA-0) 33,33,34
   33 B2(I,1)=B2(I,1)+X(NA)
      GO TO 14
   34 B2(I,1)=B2(I,1)+X(NA)*X2(NA)**LA
   14 CONTINUE
      NLCR=LCRDER+1
      CALL GAUSS2(NLOR,1,EP,SIGMA,B2,A2,NER)
```

```
      IF(NFLAG.NE.1) GO TO 97
      DO 500 JK=1,MC
      X33(JK,JAP)=A2(JK,1)
  500 CONTINUE
      JAP=JAP+1
      GO TO 96
   97 DO 501 JK=1,MC
      X31(JK)=A2(JK,1)
  501 CONTINUE
      DO 502 JK=1,IDIMT
      X3(JK)=X33(JK,JAP)
  502 CONTINUE
      DO 56 I=1,IDIMT
      XX3(I+1)=X3(I)
   56 CONTINUE
      XX3(1)=0
      DO 52 I=1,IDIMT1
      XX31(I+1)=X31(I)
   52 CONTINUE
      XX31(1)=0
      MOM=1
      MOP=1
   70 ARG=R
      CALL TVAL(RES,ARG,X3,IDIMT)
      TT3=RES
      CALL PINT(Y,IDIMY,XX3,IDIMX)
      ARG=R
      CALL PVAL(RES,ARG,Y,IDIMY)
      RR=RES
      ARG=A3
      CALL PVAL(RES,ARG,Y,IDIMY)
      AA=RES
      ARG=B3
      CALL PVAL(RES,ARG,Y,IDIMY)
      BB3=RES
      C3=RR-AA
      D3=BB3-AA
      AAA=C3/(R**2)
      BBB3=D3*(((R**2)-(A3**2))/((R**2)*((B3**2)-(A3**2))))
      F3=BBB3-AAA
      SIGMAR=((ELAS*ALPHA)/(1-POIS))*F3
      CCC=D3*(((R**2)+(A3**2))/((R**2)*((B3**2)-(A3**2))))
      G=AAA+CCC-TT3
      SIGMAT=((ELAS*ALPHA)/(1-POIS))*G
      DDD=D3*(2/((B3**2)-(A3**2)))
      H3=DDD-TT3
      SIGMAZ=((ELAS*ALPHA)/(1-POIS))*H3
      IF(R.GT.A3) GO TO 72
```

```
      EPSZ=DDC*ALPHA
      DD=D3*ALPHA*((2*(1+POIS))/((B3**2)-(A3**2)))
      U=((A3+B3)/2)*(DD-(POIS*EPSZ))
   74 ARG=R1
      CALL TVAL1(RES,ARG,X31,IDIMT1)
      TT31=RES
      CALL PINT1(Y1,IDIMY1,XX31,IDIMX1)
      ARG=R1
      CALL PVAL1(RES,ARG,Y1,IDIMY1)
      RR1=RES
      ARG=A31
      CALL PVAL1(RES,ARG,Y1,IDIMY1)
      AA1=RES
      ARG=B31
      CALL PVAL1(RES,ARG,Y1,IDIMY1)
      BB31=RES
      C31=RR1-AA1
      D31=BB31-AA1
      AAA1=C31/(R1**2)
      BBB31=D31*(((R1**2)-(A31**2))/((R1**2)*((B31**2)-(A31**2))))
      F31=BBB31-AAA1
      SIGMR1=((ELAS1*ALPHA1)/(1-POIS1))*F31
      CCC1=D31*(((R1**2)+(A31**2))/((R1**2)*((B31**2)-(A31**2))))
      G1=AAA1+CCC1-TT31
      SIGMT1=((ELAS1*ALPHA1)/(1-POIS1))*G1
      DDD1=D31*(2/((B31**2)-(A31**2)))
      H31=DDD1-TT31
      SIGMZ1=((ELAS1*ALPHA1)/(1-POIS1))*H31
      IF(R1.GT.A31) GO TO 76
      EPSZ1=DDD1*ALPHA1
      DD1=D31*ALPHA1*((2*(1+POIS1))/((B31**2)-(A31**2)))
      U1=((A31+B31)/2)*(DD1-(POIS1*EPSZ1))
      BETA=((3*(1-(POIS**2)))/((((A3+B3)/2)**2)*((B3-A3)**2)))**0.25
      DEL=(ELAS*((B3-A3)**3))/(12*(1-(POIS**2)))
      BETA1=((3*(1-(POIS1**2)))/((((A31+B31)/2)**2)*((B31-A31)**2)))
     1**0.25
      DEL1=(ELAS1*((B31-A31)**3))/(12*(1-(POIS1**2)))
      EMINF=0
      DO 1000 I=1,IDIMT
      IEP=I+1
      IE=I
 1000 EMINF=EMINF+X3(I)*((B3**IEP-A3**IEP)/IEP-(B3+A3)*(B3**I-A3**I)/
     1(IE+IE))
      EMINF=-((ELAS*ALPHA)/(1-POIS))*EMINF
      EMINF1=0
      DO 1001 I=1,IDIMT1
      IEP=I+1
      IE=I
```

```
1001 EMINF1=EMINF1+X31(I)*((B31**IEP-A31**IEP)/IEP-(B31+A31)*(B31**I-
    1A31**I)/(IE+IE))
     EMINF1=-((ELAS1*ALPHA1)/(1-POIS1))*EMINF1
     UU=U1-U-((EMINF/(2*(BETA**2)*DEL))-(EMINF1/(2*(BETA1**2)*DEL1)))
     UUU=-((EMINF/(BETA*DEL))+(EMINF1/(BETA1*DEL1)))
     P3=(1/(2*(BETA**2)*DEL))-(1/(2*(BETA1**2)*DEL1))
     Q=(1/(2*(BETA**3)*DEL))-(1/(2*(BETA1**3)*DEL1))
     S3=(1/(BETA*DEL))+(1/(BETA1*DEL1))
     W=(1/(2*(BETA**2)*DEL))+(1/(2*(BETA1**2)*DEL1))
     ENDMO=((UU*W)-(UUU*Q))/((P3*W)-(S3*Q))
     VO=((P3*UUU)-(UU*S3))/((P3*W)-(S3*Q))
     ENDP=ENDMC+EMINF
     ENDP1=ENDMC+EMINF1
     V=(VO+BETA*ENDP)/(2*(BETA**3)*DEL)
     V1=(VC+BETA1*ENDP1)/(2*(BETA1**3)*DEL1)
     BX=DATAN(VO/(2*BETA*ENDP+VO))
     BX1=DATAN(VC/(2*BETA1*ENDP1+VO))
     IF(BX.LT.0.0) BX=3.14160+BX
     IF(BX1.LT.0.0) BX1=3.14160+BX1
     XB=BX/BETA
     XB1=BX1/BETA1
     BBX=BX
     BBX1=BX1
     ENDX=(VO*(DEXP(-BBX)*DSIN(BBX)))/BETA+ENDP*(DEXP(-BBX)*
    1(DCOS(BBX)+DSIN(BBX)))
     ENDX1=(VO*(DEXP(-BBX1)*DSIN(BBX1)))/BETA1+ENDP1*(DEXP(-BBX1)*
    1(DCOS(BBX1)+DSIN(BBX1)))
     AENDX=DABS(ENDX)
     AENDMO=DABS(ENDP)
     IF(AENDX.GT.AENDMO) GO TO 615
     PRINT 630
 630 FORMAT(/,5X,'AXIAL POSITION OF MAXIMUM SIGNIFICANT STRESS FOR PIPE
    1 ONE IS AT PIPE JUNCTION.')
     SHEAR=VO
     GO TO 72
 615 PRINT 631, XB
 631 FORMAT(/,5X,'AXIAL POSITION OF MAXIMUM SIGNIFICANT STRESS FOR PIPE
    1 ONE IS AT Z =',F10.5,'INCHES.')
     SHEAR=DEXP(-BBX)*((VO*(DCOS(BBX)-DSIN(BBX)))-(2*BETA*ENDP*
    1DSIN(BBX)))
     ENDP=ENDX
  72 SIGZX=-(12*ENDP*RY)/((B3-A3)**3)
     SIGTX=(ELAS*V)/((A3+B3)/2)+(POIS*SIGZX)
     PRINT 89
  89 FORMAT(/,3X,' RADIUS  ',2X,' TEMP.  ',4X,'  RADIAL STRESS ',4X,'
    1CIRC. STRESS  ',4X,'AXIAL  STRESS   ',2X,'DIS. CIRC. STRESS ',4X,'
    2DIS. AXIAL STRESS')
     WRITE(6,79) R,TT3,SIGMAR,SIGMAT,SIGMAZ,SIGTX,SIGZX
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```
      SIGR=SIGMAR
      SIGT=SIGMAT+SIGTX
      SIGZ=SIGMAZ+SIGZX
      TAU=((3*SHEAR)*(1-((2*RY)/(B3-A3))**2))/(2*(B3-A3))
      ST1=SIGT
      ST2=((SIGZ+SIGR)/2)+(((SIGZ-SIGR)/2)**2+TAU**2)**C.5
      ST3=((SIGZ+SIGR)/2)-(((SIGZ-SIGR)/2)**2+TAU**2)**0.5
      RT=DABS(ST1-ST2)
      TZ=DABS(ST2-ST3)
      ZR=DABS(ST3-ST1)
      BIG=RT
      IF(TZ.GT.BIG) BIG=TZ
      IF(ZR.GT.BIG) BIG=ZR
      PRINT 620, RT
  620 FORMAT(/,5X,'TYPE 1 =',F15.6,' PSI.')
      PRINT 621, TZ
  621 FORMAT(5X,'TYPE 2 =',F15.6,' PSI.')
      PRINT 622, ZR
  622 FORMAT(5X,'TYPE 3 =',F15.6,' PSI.')
      PRINT 623, BIG
  623 FORMAT(/,10X,'SIGNIFICANT STRESS IS',F15.6,' PSI.')
      STRESS(MCM)=BIG
      RAD(MCM)=R
      IF(NSTAR.NE.1) GO TO 900
      IF(R.NE.A3) GC TO 75
      R=B3
      RY=((B3-A3)/2)
      MCM=MCM+1
      GO TC 70
  900 RY=RY+0.010
      R=R+0.010
      MOM=MCM+1
      IF(R.LE.B3) GC TO 70
      IF(R.GT.B3) GC TO 75
   79 FORMAT(2F10.3,5F20.6)
   75 AENDX1=DABS(ENDX1)
      AENDMO=DABS(ENDP1)
      IF(AENDX1.GT.AENDMO) GO TO 616
      PRINT 632
  632 FORMAT(/,5X,'AXIAL POSITION OF MAXIMUM SIGNIFICANT STRESS FCR PIPE
     1 TWO IS AT PIPE JUNCTION.')
      SHEAR1=VO
      GO TO 76
  616 PRINT 633, XB1
  633 FORMAT(/,5X,'AXIAL POSITION OF MAXIMUM SIGNIFICANT STRESS FOR PIPE
     1 TWO IS AT Z =',F10.5,'INCHES.')
      SHEAR1=DEXP(-BBX1)*((VO*(DCOS(BBX1)-DSIN(BBX1)))-(2*BETA1*
     1ENDP1*DSIN(BBX1)))
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```
      ENDP1=ENDX1
76    SIGZX1=-(12*ENDP1*RY1)/((B31-A31)**3)
      SIGTX1=(ELAS1*V1)/((A31+B31)/2)+(POIS1*SIGZX1)
      PRINT 89
      WRITE(6,67) R1,TT31,SIGMR1,SIGMT1,SIGMZ1,SIGTX1,SIGZX1
      SIGR1=SIGMR1
      SIGT1=SIGMT1+SIGTX1
      SIGZ1=SIGMZ1+SIGZX1
      TAU1=((3*SHEAR1)*(1-((2*RY1)/(B31-A31))**2))/(2*(B31-A31))
      ST11=SIGT1
      ST21=((SIGZ1+SIGR1)/2)+(((SIGZ1-SIGR1)/2)**2+TAU1**2)**0.5
      ST31=((SIGZ1+SIGR1)/2)-(((SIGZ1-SIGR1)/2)**2+TAU1**2)**0.5
      RT1=DABS(ST11-ST21)
      TZ1=DABS(ST21-ST31)
      ZR1=DABS(ST31-ST11)
      BIG1=RT1
      IF(TZ1.GT.BIG1) BIG1=TZ1
      IF(ZR1.GT.BIG1) BIG1=ZR1
      PRINT 620, RT1
      PRINT 621, TZ1
      PRINT 622, ZR1
      PRINT 623, BIG1
      STRES1(MOP)=BIG1
      RAD1(MOP)=R1
      IF(NSTAR.NE.1) GO TO 901
      IF(R1.NE.A31) GO TO 64
      R1=B31
      RY1=((B31-A31)/2)
      MOP=MOP+1
      GO TO 74
901   RY1=RY1+0.010
      R1=R1+0.010
      MOP=MOP+1
      IF(R1.LE.B31) GO TO 74
      IF(R1.GT.B31) GO TO 64
67    FORMAT(2F10.3,5F20.6)
64    BEST=STRESS(1)
      ROCK=RAD(1)
      DO 607 I=2,MOM
      IF(STRESS(I).GT.BEST) GO TO 643
      GO TO 607
643   BEST=STRESS(I)
      ROCK=RAD(I)
607   CONTINUE
      SIG(JAP)=BEST
      TIME(JAP)=T
      BUZZ(JAP)=ROCK
      BEST1=STRES1(1)
```

```
      ROCK1=RAD1(1)
      DO 608 I=2,MCP
      IF(STRES1(I).GT.BEST1) GO TO 644
      GO TO 608
  644 BEST1=STRES1(I)
      ROCK1=RAD1(I)
  608 CONTINUE
      SIG1(JAP)=BEST1
      BUZZ1(JAP)=ROCK1
      JAP=JAP+1
   96 FRAC=.5D+0
      X(1)=FRAC*X(1)+(1.-FRAC)*FLU
      NCOUNT=NCOUNT+1
      DO 45 I=1,N
      AL(I)=K(X(I))/(SPHT(X(I))*DENS(X(I)))
   45 KAY(I)=K(X(I))*A(I)
      SURTEM=X(1)
      H=SURFCO(SURTEM)
      HO=0.
      XX(1)=X(1)+(AL(1)*P/KAY(1))*(2.*H*DR*A(1)*(FLU-X(1))+(KAY(1)+
     1KAY(2))*(X(2)-X(1)))
      DO 46 I=2,NM
      IP=I+1
      IM=I-1
   46 XX(I)=X(I)+(.5*AL(I)*P/KAY(I))*((KAY(I)+KAY(IM))*(X(IM)-X(I))+
     1(KAY(I)+KAY(IP))*(X(IP)-X(I)))
      XX(N)=X(N)+(AL(N)*P/KAY(N))*(2.*HO*DR*A(N)*(XFO-X(N))+(KAY(N)+
     1KAY(NM))*(X(NM)-X(N)))
      DO 47 I=1,N
   47 X(I)=XX(I)
      T=T+DT
      Z=-.5*(X(1)*A(1)+X(N)*A(N))
      DO 48 I=1,N
   48 Z=Z+A(I)*X(I)
      XAVG=Z/DNOM
      DELTA1=X(1)-X(N)
      DELTA2=(X(1)+X(N))/2.-XAVG
      DETT1=DABS(DELTA1)
      DETT2=DABS(DELTA2)
      IF(DETT1.LE.FELTA1) GO TO 49
      TT1=T
      FELTA1=DETT1
      GLUG1=DETT2
   49 IF(DETT2.LE.FELTA2) GO TO 50
      TT2=T
      FELTA2=DETT2
      GLUG2=DETT1
   50 CONTINUE
```

```
      IF(NCCUNT.NE.NPRINT) GC TC 55
      NCOUNT=0
      CALL FLUTEM(T,FLU,ZED)
      PRINT 41,T,DELTA1,DELTA2,XAVG, FLU,(X(I),I=1,N,NN)
      XF=FLU
   55 IF(T.LT.TMAX) GC TC 44
      IF(NFLAG.EQ.1) GC TC 610
      JAPM=JAP-1
      SSIG=SIG(1)
      TIMA=TIME(1)
      GAKA=BUZZ(1)
      DO 611 I=2,JAPM
      IF(SIG(I).GT.SSIG) GO TC 641
      GO TC 611
  641 SSIG=SIG(I)
      TIMA=TIME(I)
      GAKA=BUZZ(I)
  611 CONTINUE
      SSIG1=SIG1(1)
      TIMA1=TIME(1)
      GAKA1=BUZZ1(1)
      DO 612 I=2,JAPM
      IF(SIG1(I).GT.SSIG1) GO TO 642
      GO TO 612
  642 SSIG1=SIG1(I)
      TIMA1=TIME(I)
      GAKA1=BUZZ1(I)
  612 CONTINUE
      PRINT 613, TIMA,SSIG,GAKA
  613 FORMAT(//,5X,'AT TIME EQUALS',F10.3,' SECONDS, THE MAXIMUM SIGNIFI
     1CANT STRESS FOR PIPE CNE IS',F15.6,' PSI.',/,5X,'THE RACIAL COORDI
     2NATE OF THIS STRESS IS AT R =',F10.3,' INCHES.')
      PRINT 614, TIMA1,SSIG1,GAKA1
  614 FORMAT(//,5X,'AT TIME EQUALS',F10.3,' SECONDS, THE MAXIMUM SIGNIFI
     1CANT STRESS FOR PIPE TWC IS',F15.6,' PSI.',/,5X,'THE RACIAL COORDI
     2NATE OF THIS STRESS IS AT R =',F10.3,' INCHES.')
  610 T=T-DT
      PRINT 51,T,TT1,FELTA1,GLUG1,TT2,GLUG2,FELTA2
   51 FORMAT(//,5X,'END OF PROBLEM AT TIME EQUALS ',F8.2,' SECONDS.',
     1//,5X,'AT TIME EQUALS ',F8.2,' SECONDS, DELTA-TEE-ONE (MAXIMUM) ',
     2'IS ',F8.2,' DEGREES,',/,5X,'AND DELTA-TEE-TWO IS ',F8.2,' DEGRE',
     3'ES.',//,5X,'AT TIME EQUALS ',F8.2,' SECONDS, DELTA-TEE-ONE IS ',
     4 F8.2,' DEGREES',/,5X,'AND DELTA-TEE-TWO (MAXIMUM) IS ',F8.2,
     5 'DEGREES.')
      IF(NFLAG.EQ.1) GO TC 280
 5000 CONTINUE
      STOP
      END
```

```
      SUBROUTINE GAUSS2(NLOR,M,EP,A22,B2,X22,NER)
      IMPLICIT REAL*8 (A-H,K,C-Z), INTEGER (I,J,L,M,N)
      DIMENSION A22(10,30),B2(10,20),X22(10,20)
      NPM=NLOR+M
      DO 2 IK=1,M
      I=NLOR+IK
      DO 2 J=1,NLCR
    2 A22(J,I)=B2(J,IK)
   10 DO 34 L=1,NLCR
      IKP=0
      Z2=0.0
      DO 12 IK=L,NLCR
      IF(Z2-DABS(A22(IK,L))) 11,12,12
   11 Z2=DABS(A22(IK,L))
      IKP=IK
   12 CONTINUE
      IF(L-IKP) 13,20,20
   13 DO 14 J=L,NPM
      Z2=A22(L,J)
      A22(L,J)=A22(IKP,J)
   14 A22(IKP,J)=Z2
   20 IF(DABS(A22(L,L))-EP) 50,50,30
   30 IF(L-NLCR) 31,40,40
   31 LP1=L+1
      DO 34 IK=LP1,NLCR
      IF(A22(IK,L)) 32,34,32
   32 RATIO=A22(IK,L)/A22(L,L)
      DO 33 J=LP1,NPM
   33 A22(IK,J)=A22(IK,J)-RATIO*A22(L,J)
   34 CONTINUE
   40 DO 43 I=1,NLCR
      III=NLOR+1-I
      DO 43 J=1,M
      JPN=J+NLCR
      S=0.0
      IF(III-NLOR) 41,43,43
   41 IIP1=III+1
      DO 42 IK=IIP1,NLOR
   42 S=S+A22(III,IK)*X22(IK,J)
   43 X22(III,J)=(A22(III,JPN)-S)/A22(III,III)
      NER=1
      RETURN
   50 NER=2
      END
```

```
      SUBROUTINE FLUTEM(ARG1,ARG2,ARG3)
      IMPLICIT REAL*8 (A-H,K,O-Z), INTEGER (I,J,L,M,N)
      DIMENSION TEM(30),FLUI(30),FLUO(30),A2(10,20)
      COMMON TEM,FLUI,FLUO,A2,MO
      J=1
      IF(ARG1.LT.TEM(J)) GO TO 81
   80 J=J+1
      IF(ARG1.GT.TEM(J)) GO TO 80
      JJ=J-1
      DONE=(TEM(J)-ARG1)/(TEM(J)-TEM(JJ))
      DTWO=(ARG1-TEM(JJ))/(TEM(J)-TEM(JJ))
      ARG2 =FLUI(J)*DTWO+FLUI(JJ)*DONE
      ARG3 =FLUO(J)*DTWO+FLUO(JJ)*DONE
      RETURN
   81 PRINT 82, ARG1, TEM(1)
   82 FORMAT(7X,'ERROR IN FLUID TEMPERATURE CALCULATION',/,
     17X,'SUBROUTINE INTERROGATED WITH ARG1 = ',F12.6,':',/,
     27X,'SMALLEST TABULATED VALUE OF TIME = ',F12.6,' SECONDS.')
      STOP
      END


      SUBROUTINE PINT(Y,IDIMY,XX3,IDIMX)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION XX3(100),Y(100)
      IDIMY=IDIMX+1
      Y(1)=0
      IF(IDIMX)1,1,2
    1 RETURN
    2 EXPT=1
      DO 3 I=2,IDIMY
      Y(I)=XX3(I-1)/EXPT
    3 EXPT=EXPT+1
      GO TO 1
      END
```

```
      SUBROUTINE PVAL(RES,ARG,Y,IDIMY)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION Y(100)
      RES=0.
      J=IDIMY
    4 IF(J)6,6,5
    5 RES=RES*ARG+Y(J)
      J=J-1
      GO TO 4
    6 RETURN
      END


      SUBROUTINE TVAL(RES,ARG,X3,IDIMT)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION X3(100)
      RES=0.
      J=IDIMT
   11 IF(J) 13,13,12
   12 RES=RES*ARG+X3(J)
      J=J-1
      GO TO 11
   13 RETURN
      END


      SUBROUTINE PINT1(Y1,IDIMY1,XX31,IDIMX1)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION XX31(100),Y1(100)
      IDIMY1=IDIMX1+1
      Y1(1)=0
      IF(IDIMX1)14,14,15
   14 RETURN
   15 EXPT=1
      DO 16 I=2,IDIMY1
      Y1(I)=XX31(I-1)/EXPT
   16 EXPT=EXPT+1
      GO TO 14
      END
```

```
      SUBROUTINE PVAL1(RES,ARG,Y1,IDIMY1)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION Y1(100)
      RES=0.
      J=IDIMY1
   19 IF(J)17,17,18
   18 RES=RES*ARG+Y1(J)
      J=J-1
      GO TO 19
   17 RETURN
      END



      SUBROUTINE TVAL1(RES,ARG,X31,IDIMT1)
      IMPLICIT REAL*8 (A-H,K,O-Z),INTEGER(I,J,L,M,N)
      DIMENSION X31(100)
      RES=0.
      J=IDIMT1
   21 IF(J)23,23,22
   22 RES=RES*ARG+X31(J)
      J=J-1
      GO TO 21
   23 RETURN
      END
```

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13 ABSTRACT

A method is devised for the transient-state thermal stress analysis of two pipes, joined butt to butt and subjected to rapid or sudden change of internal fluid temperature. Although it is assumed that there is symmetry about the common pipe axis, the properties of the materials as well as the thickness of each pipe may be different. Then, given a specified time-temperature relationship for the internal fluid, over a specified problem time, the maximum stress encountered may be obtained. A digital computer program is appended for the solution of such problems.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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COMPUTER THERMAL STRESS ANALYSIS						
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THERMAL STRESSES IN PIPES						
THERMAL STRESSES IN A BUTT JOINT IN PIPING						